Naval Research Laboratory

Washington, DC 20375-5000 NRL Report 8984 June 20, 1986

UNCLASSIFIED

Degradation in Performance of Adaptive Null-Steering Antennas

WALTER K. KAHN

Airborne Radar Branch
Radar Division

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Report 8984	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b. OFFICE SYMBOL *(If applicable)* Code 5360K	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS *(City, State, and ZIP Code)* Washington, DC 20375-5000	7b. ADDRESS *(City, State, and ZIP Code)*
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Naval Air Systems Command 8b. OFFICE SYMBOL *(If applicable)*	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS *(City, State, and ZIP Code)* Washington, DC 20361	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 62712N PROJECT NO. TASK NO. WF-12-131-701 WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*

Degradation in Performance of Adaptive Null-Steering Antennas

12. PERSONAL AUTHOR(S)

Kahn, Walter K.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT *(Year, Month, Day)*	15. PAGE COUNT
Interim	FROM ______ TO ______	1986 June 20	12

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
FIELD	GROUP	SUB-GROUP	Phased arrays Adaptive arrays Null forming arrays

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

The degradation in the overall main beam gain performance capability of a phased array, which necessarily accompanies the required formation of pattern of pattern nulls, is quantified in a simple way. The result is in accord with intuitive expectation.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Walter K. Kahn	22b. TELEPHONE *(Include Area Code)* (202) 767-2569 22c. OFFICE SYMBOL 5360K

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

CONTENTS

DEGRADATION IN PERFORMANCE OF ADAPTIVE NULL-STEERING ANTENNAS

INTRODUCTION

This report quantifies the degradation in the overall main beam gain performance capability of any phased array antenna which necessarily accompanies the formation of pattern nulls in a number of directions. The adaptive formation of pattern nulls constitutes one strategy through which a radar may seek to suppress effects of stand-off jammers. However, an array comprising a finite number of elementary radiators disposes of only a finite number of degrees of freedom. Thus it has long been recognized that as the number of nulls required approaches the number of degrees of freedom, antenna performance deteriorates. Here, this fact is given a simple quantitative formulation very much in accord with our intuitive expectation.

PERFORMANCE

The primary function of a phased array antenna is to form a high-gain beam along any direction within some (limited) scan volume. From this standpoint, the performance of a phased array is summarized by the envelope gain function of the array $G_{\max}(\Omega)$; other pattern characteristics such as (peak) sidelobe level are not considered here. This function specifies the maximum realizable gain from the complete array in any given direction $\Omega \equiv (\theta,\phi)$, when the elements of the array are excited by real generators matched to their waveguide leads with optimum amplitude and phase for gain in that particular direction. The envelope gain function is, of course, quite different from the conventional array gain-pattern function corresponding to any one particular scan angle, i.e., fixed distribution of excitation. Figure 1 contrasts the gain envelope and conventional gain-pattern functions. Under certain conditions, we can relate the number of antenna elements in the array and array performance as specified by the envelope gain function [1,2].

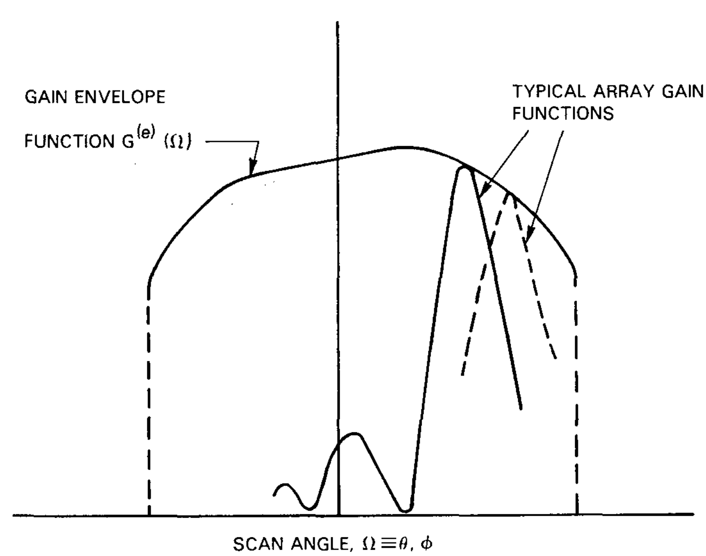

Fig. 1 — Schematic representation of gain-envelope function and typical antenna array gain functions for fixed scan angles (fixed excitation)

Manuscript approved March 10, 1986.

We assume the elementary antennas of the array to be essentially lossless (Lorentz) reciprocal components; dissipation occurs only in the internal impedance loads of the generators exciting each antenna (or feed network). See Fig. 2. Further, we shall assume that at any given operating frequency:

- a fixed total available power can be distributed arbitrarily among the individual radiating elements;
- each element is equipped with a phase shifter;
- in any given direction, all elementary antennas radiate fields with the same polarization.

The last assumption permits us to use scalar theory. Although there is no requirement that all the elementary antennas be otherwise similar, this is normally the case. This then assures that the third assumption is satisfied, at least to first order. The third assumption is valid exactly if the elementary antennas have identical field patterns in the open-circuited array environment.

Under the foregoing conditions, we may show [see appendix] that

$$N = \frac{1}{\eta} \frac{1}{4\pi} \iint_{\text{scan volume}} G_{\max}^{(e)}(\Omega')\, d\Omega', \tag{2.1}$$

where

N is the number of elements in the array;

η is the average element efficiency;

$G_{\max}^{(e)}(\Omega)$ is the maximal gain-envelope function;

Ω is the bearing in space equivalent to the spherical coordinates θ, ϕ; and

$d\Omega$ is the differential element of solid angle $(\sin\theta) d\theta\, d\phi$.

The average element efficiency [3,4] is given by

$$\eta = \frac{1}{N} \sum \eta_m, \tag{2.2}$$

where η_m, the efficiency associated with the mth element of the array, is the ratio of the power radiated from the entire array when only the mth element is excited to the power available from the generator exciting that mth element.

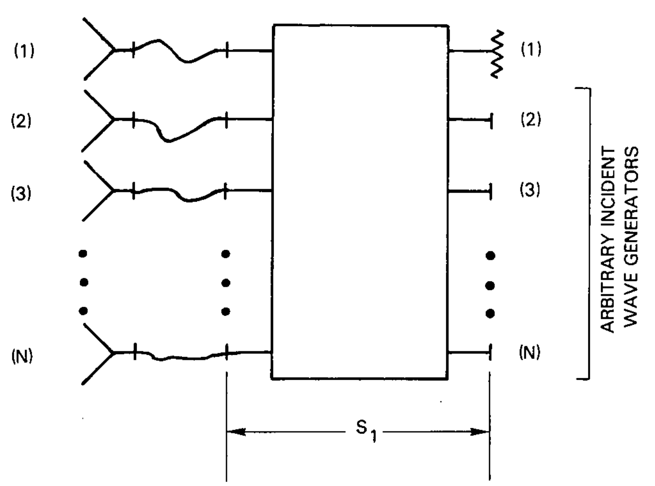

Fig. 2 — Arbitrary incident wave excitation at $N - 1$ ports of the transparent feed network S maintains the null in direction Ω_1

The element efficiency just defined is a measure of mutual coupling.

$$\eta_m = \frac{P_m \text{ (radiated)}}{P_m \text{ (available)}} = \frac{P_m \text{ (available)} - P_m \text{ (reflected)}}{P_m \text{ (available)}},$$

$$= 1 - \sum_{n=1}^{N} |S_{nm}|^2. \tag{2.3}$$

The coefficients S_{nm} are the normalized scattering coefficients describing coupling among the elements of the array.

$$b_n = \sum_{m=1}^{N} S_{nm} a_m \tag{2.4a}$$

$$2\sqrt{R_g}\, a_m = V_m + Z_g I_m \tag{2.4b}$$

$$2\sqrt{R_g}\, b_m = V_m - Z_g^* I_m \tag{2.4c}$$

$$Z_g = R_g + jX_g. \tag{2.4d}$$

Evidently the stronger the mutual coupling the smaller the element efficiency η. The smaller the element efficiency, the larger is the number of antenna array elements N required for a given gain envelope performance. It is remarkable that the number of antenna elements required does not explicitly depend on the gain functions of these individual array elements, and (since $\eta \leqslant 1$) that there exists an absolute minimum value of N.

EFFECT OF THE NULL-FORMING CONSTRAINT

Consider a specified antenna array where the gain performance is summarized by the gain envelope function $G_0^{(e)}(\Omega)$. This particular array then must comprise N

$$N \geqslant \frac{1}{4\pi\eta} \iint G_0^{(e)}(\Omega')\, d\Omega' \tag{3.1}$$

elementary antennas. We now propose to employ this relation, in reverse as it were, as a constraint governing the performance of the array under the assumption that the array must form nulls in certain prescribed directions.

Suppose that a radiation null is demanded in the direction $\Omega_1 \equiv (\theta_1, \phi_1)$. We will construct a transparent $2N$-port network which, when connected in tandem with all the elements of the array, will effect the following result: arbitrary excitation of the array at $N-1$ of the input ports to this network will maintain the prescribed null. A lossless reciprocal $2N$-port network has been termed "transparent" when waves incident at one set of N ports do not cause reflected waves to emerge at any of those N ports. Let $f_n(\Omega_1)$ be proportional to (a component of) the far field in the direction Ω_1 obtained when the nth element of the original array is excited by a unit incident wave amplitude, $a_n = 1$, in the environment of all the remaining elements terminated in loads, Z_g.

If necessary, we add lengths of transmission line at any port so that the generally complex quantities $f_n^{(1)}(\Omega_1)$ are all real and positive. The $f_n^{(1)}(\Omega_1)$ can be ordered into a column vector $\mathbf{f}^{(1)}$ ($\sim$ denotes the transpose):

$$\tilde{\mathbf{f}}^{(1)} = \left[f_1^{(1)}(\Omega_1) f_2^{(1)}(\Omega_1) \ldots f_N^{(1)}(\Omega_1) \right]. \tag{3.2}$$

As shown in the appendix, an excitation of the array with incident wave amplitudes proportional to $\mathbf{f}^{(1)}$ realizes the maximum gain in the direction Ω_1 obtainable with given array and given real generators. Any excitation of the array orthogonal to $\mathbf{f}^{(1)}$ preserves a null in the direction Ω_1.

The transparent network that physically separates out $N-1$ ports that may be excited by arbitrary incident waves without disturbing the null in the direction Ω_1 is constructed as follows. Form the vector

$$\mathbf{h}_1^{(1)} = \mathbf{f}^{(1)} (\tilde{\mathbf{f}}^{(1)}\mathbf{f}^{(1)})^{-1}. \tag{3.3}$$

By means of the Gram-Schmidt process, provide N-1 additional orthornormal vectors $\mathbf{h}_2^{(1)}, \mathbf{h}_3^{(1)}, \ldots,$ $\mathbf{h}_N^{(1)}$ spanning the subspace orthogonal to $\mathbf{h}_1^{(1)}$, and define the transformation matrix

$$T_1 = \left[\mathbf{h}_1^{(1)}\ \mathbf{h}_2^{(1)} \ldots \mathbf{h}_N^{(1)}\right]. \tag{3.4}$$

The scattering matrix

$$S_1 = \left[\begin{array}{c|c} 0 & \tilde{T}_1 \\ \hline T_1 & 0 \end{array}\right] \tag{3.5}$$

then clearly represents a lossless reciprocal $2N$-port which, on account of the properties associated with zero submatrices on the principal diagonal, has been termed "transparent." Ports $N+1$ to $2N$ are to be connected to antenna elements 1 to N, respectively. By this construction, shown in Fig. 2, arbitrary excitations of ports 2 to N of the transparent network (i.e., no excitation at port 1) will preserve the required null.

Using the realized gain functions corresponding to excitation of these same ports 2 to N, we compute a modified gain-envelope function

$$G_1^{(e)}(\Omega) \leqslant G_0^{(e)}(\Omega). \tag{3.6}$$

The essence of this analysis is to observe that this new gain envelope function must satisfy the integral constraint

$$N-1 \geqslant \frac{1}{4\pi\eta} \underset{\text{scan volume}}{\iint} G_1^{(e)}(\Omega')d\Omega'. \tag{3.7}$$

It is important to observe that $\eta \leqslant 1$, which accounts for the mutual coupling present in the originally specified array, is unaffected (invariant) when the array is cascaded with a *transparent* $2N$-port network. This is demonstrated in the appendix of this report.

Suppose now that in addition a second null, in the direction $\Omega_2 \equiv (\theta_2, \phi_2)$ is required. Let $f_n^{(2)}(\Omega_2)$ be proportional to a component of the far field radiated in the direction Ω_2 when the nth port of the transparent network found above (Fig. 2) is excited by a unit incident wave amplitude.

If necessary, we again add lengths of transmission line at any port such that the generally complex quantities $f_n^{(2)}(\Omega_2)$ are all real and positive. It is possible (though unlikely) that

$$\left[f_1^{(2)}(\Omega_2)\ f_2^{(2)}(\Omega_2) \ldots f_N^{(2)}(\Omega_N)\right] \tag{3.8}$$

is of the form

$$\left[f_1^{(2)}(\Omega_2)\ 0 \quad 0 \ \ldots\ 0\right],$$

i.e., a null in the direction Ω_2 is automatically assured by excluding excitation at port 1.

In the more likely general circumstance,

$$\tilde{\mathbf{f}}^{(2)} = \left[0\ f_2^{(2)}\ (\Omega_2)\ f_2^{(2)}\ (\Omega_3) \ldots f_N^{(2)}\ (\Omega_2)\right] \tag{3.9}$$

is not the zero vector, and excitation proportional to $\mathbf{f}^{(2)}$ realizes the gain $G_1^{(e)}(\Omega_2)$ in the direction Ω_2.

We now form the vector

$$\mathbf{h}^{(2)} = \mathbf{f}^{(2)}\,(\tilde{\mathbf{f}}^{(2)}\mathbf{f})^{-1}. \tag{3.10}$$

Using the Gram-Schmidt process, we again provide $N-2$ additional orthonormal vectors $\mathbf{h}_3^{(2)}, \mathbf{h}_3^{(2)}, \ldots, \mathbf{h}_N^{(2)}$ spanning the subspace orthogonal to that spanned by $\mathbf{h}_1^{(1)}$, $\mathbf{h}_2^{(2)}$, and define the transformation matrix

$$T_2 = [\boldsymbol{\delta}_1\ \mathbf{h}_2^{(2)}\ \mathbf{h}_3^{(2)}\ \ldots\ \mathbf{h}_N^{(2)}]. \tag{3.11}$$

The vector $\boldsymbol{\delta}_n$ has an nth element equal to unity, and all other elements are zero. The scattering matrix

$$S_2 = \left[\begin{array}{c|c} 0 & \tilde{T}_2 \\ \hline T_2 & 0 \end{array}\right] \tag{3.12}$$

represents a transparent $2N$-port network. Actually $2N$-port notation is retained only for convenience in bookkeeping associated with port numbers. The $\tilde{\boldsymbol{\delta}}_1$ row and $\boldsymbol{\delta}_1$ column appearing in $\tilde{T}_2$ and T_2 are simply interpreted as requiring no further connection to port number 1 of the first transparent $2N$-port. Effectively, a new $2(N-1)$ port is connected to the array. Figure 3 shows the result. By construction, arbitrary excitation of ports 3 to N of the second transparent network (i.e., no excitation at ports 1 and 2) will preserve the required two nulls. Using the realized gain functions corresponding to excitations of these $N-2$ ports, we compute a further modified gain envelope function

$$G_2^{(e)}(\Omega) \leqslant G_1^{(e)}(\Omega) \tag{3.13}$$

that satisfies the constraint

$$N-2 \geqslant \frac{1}{4\pi\eta} \iint_{\text{scan volume}} G_2^{(e)}(\Omega')\,d\Omega'. \tag{3.14}$$

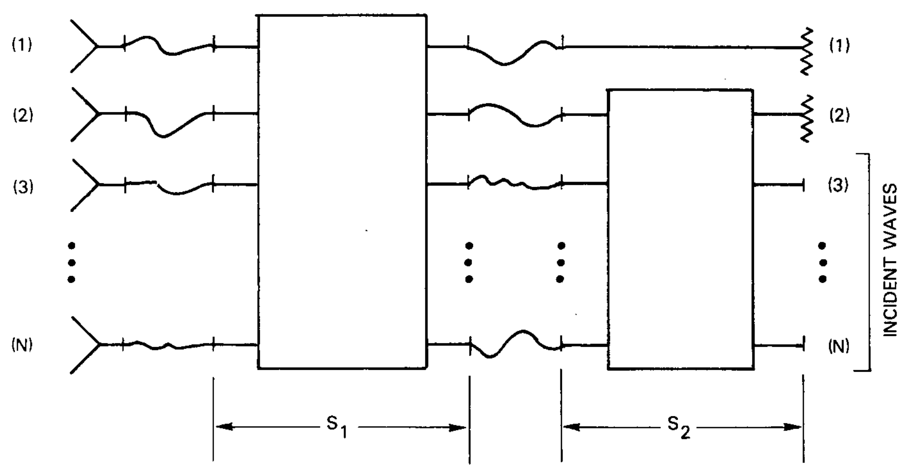

Fig. 3 — Arbitrary incident wave excitation at $N-2$ ports of the transparent feed network S_2 maintains the nulls in directions Ω_1 and Ω_2

Evidently this process can be repeated for additional prescribed nulls. Should it be repeated N times, the effect on array performance would clearly be catastrophic.

However, at every intermediate iteration of this process, the number of ports that can be excited while maintaining the prescribed (independent) nulls is reduced by one. Thus M independent prescribed nulls effectively reduces an N element array to an $N-M$ array of elements (ports) that can be arbitrarily excited. Consequently the gain envelope for the array, constrained to retain the prescribed nulls, must satisfy

$$N-M \geqslant \frac{1}{4\pi\eta} \iint\limits_{\text{scan volume}} G_M^{(e)}(\Omega')\,d\Omega'. \tag{3.15}$$

The gain envelope (performance) $G_M^{(e)}$, *on the average*, is therefore necessarily reduced relative to the performance attainable without null constraints (2.5).

CONCLUSION

It has been shown that the requirement for an N-element array antenna (adaptively) to form nulls in a number of directions $\Omega_1, \Omega_2, \ldots, \Omega_M$, $M \leqslant N$, necessarily degrades the achievable performance on the average from the array. The qualification "on the average," of course, refers to the integral nature of the constraints (3.15).

ACKNOWLEDGMENT

The author is grateful to Dr. T.L. ap Rhys for his interest and valuable comments.

REFERENCES

1. J.J. Stangel, "A Basic Theorem Concerning the Electronics Scanning Capabilities of Antennas," 1974 URSI Spring Meeting, Georgia Institute of Technology, Atlanta, GA. Digest Reference VI, 3-8, p. 50.

2. W. Wasylkiwskyj and W.K. Kahn, "Efficiency as a Measure of Size of a Phased Array Antenna," *IEEE Trans. Antennas Propagat.* **AP-21**, 879-884, Nov. 1973.

3. P.W. Hannan, "The Element-Gain Paradox for a Phased Array Antenna," *IEEE Trans. Antennas Propagat.* **AP-12**, 423-433, July 1954.

4. W.K. Kahn, "Active Reflection Coefficient and Element Efficiency in Arbitrary Antenna Arrays," *IEEE Trans. Antennas Propagat.* **AP-17**, 653-654, Sept. 1969.

Appendix

DEFINITION AND PHYSICAL SIGNIFICANCE OF ELEMENT EFFICIENCY

The efficiency of the mth elementary antenna within the environment of an entire array of N terminated lossless antenna elements may be defined [A1] as

$$\eta_m = \frac{P_m\,(\text{radiated})}{P_m\,(\text{available})} = 1 - \frac{P_m\,(\text{reflected})}{P_m\,(\text{available})} \tag{A1}$$

where P_m (radiated) is the power radiated into space when the mth element only is excited, P_m (available) is the power available at the mth element, and P_m (reflected) is the power reflected back into the single excited element plus the power reflected (coupled) into all the remaining array elements that are not excited. Since for power per steradian $P(\Omega)$ at any angle bearing $\Omega \rightarrow (\theta,\phi)$ the gain functions, the directive gain g_d, and the realized gain g_r are, respectively, [A1]

$$g_d(\Omega) = 4\pi \,\frac{P(\Omega)}{P(\text{radiated})}, \tag{A2a}$$

$$g_r(\Omega) = 4\pi \,\frac{P(\Omega)}{P(\text{available})}, \tag{A2b}$$

and one has, for the mth element,

$$\eta_m = \frac{g_{rm}(\Omega)}{g_{dm}(\Omega)}. \tag{A3}$$

Finally, when the coupling among the elements of the array is described at some fixed frequency by a normalized scattering matrix $S = [S_{nm}]$, relating incident-wave amplitudes a_m to corresponding reflected-wave amplitudes b_n at each antenna port,

$$b_n = \sum_{m=1}^{N} S_{nm}\, a_m, \quad m = 1,2,\ldots,N \tag{A4}$$

the efficiency of the mth element given in Eq. (A1) may be written as

$$\eta_m = 1 - \sum_{n=1}^{N} |s_{nm}|^2. \tag{A5}$$

The maximum gain attainable from the complete array in any given direction Ω', when elements are individually excited by matched generators in the optimal fashion to yield the maximum gain in that direction, is given in terms of realized gain of the individual elements [A2] by

$$G_{\max}(\Omega') = \sum_{m=1}^{N} g_{rm}(\Omega'). \tag{A6}$$

This is most readily verified as follows. By definition the realized gain of the array in the direction Ω_0 is

$$G_r(\Omega_0) = \frac{\left|\sum_{m=1}^{N} f_m(\Omega_0)\, a_m\right|^2}{P\ (\text{available})/4\pi}, \tag{A7}$$

where the reference planes have been chosen so that the $f_n(\Omega_0) = \sqrt{g_{rn}(\Omega_0)}$ are real and positive. The numerator may be considered as the scalar product of two vectors $\tilde{\mathbf{f}} \cdot \mathbf{a}$ which is maximized when $\mathbf{a}$ is parallel to $\mathbf{f}$. Setting

$$a_m = \alpha\, f_m(\Omega_0) \tag{A8}$$

we have

$$P\ (\text{available}) = \sum_{m=1}^{N} |a_m|^2 = |\alpha|^2 \sum_{m=1}^{N} |f_m(\Omega_0)|^2. \tag{A9}$$

Substituting in Eq. (A7) we obtain Eq. (A6). The mean value of Eq. (A6) averaged over all angles Ω' in space yields

$$\frac{1}{4\pi} \iint_{\text{sphere}} G_{\max}(\Omega')\, d\Omega' = \sum_{m-1}^{N} \eta_m. \tag{A10}$$

Thus, in terms of the mean efficiency averaged over all the elements in the array [A2],

$$\eta = \frac{1}{N} \sum_{m-1}^{N} \eta_m \tag{A11}$$

one finds that

$$N = \frac{1}{\eta} \frac{1}{4\pi} \iint_{\text{sphere}} G_{\max}(\Omega')\, d\Omega'. \tag{A12}$$

This provides a valuable estimate of the number of antenna elements that are required to realize a desired "gain envelope" for an array. Although the actual value of η for a particular array may not be known, an upper bound may be established. Such upper bounds on η, termed "ideal" efficiencies η_i, have been computed for circular cylindrical arrays [A3] and various regular infinite linear and planar arrays [A1, A4]. In these two array configurations all elements occupy equivalent positions, and, therefore, by symmetry, $\eta_m = \eta$, for all m. The tabulated values of the bound η_i may be inserted in Eq. (A12) to obtain a minimum estimate for N.

Finally, if the array with scattering matrix $S = [S_{mn}]$ is connected in tandem with a transparent $2N$ port

$$S = \left[\begin{array}{c|c} 0 & \tilde{T} \\ \hline T & 0 \end{array}\right] \tag{A13}$$

where T is a real orthogonal matrix, $\tilde{T} = T^{-1}$, then the scattering matrix at the input to the resulting network is $S' = \tilde{T}ST$. The average element efficiency η is given by the matrix expression [A2]

$$\eta = \frac{1}{N} \text{trace } \{S^+ S\}, \tag{A14}$$

where $S^+ = \tilde{S}^*$. This trace is invariant under the transformation of S to S'; in fact $S^+S' = (\tilde{T}ST)^+ \tilde{T}ST = \tilde{T}S^+T\tilde{T}ST = S^+S$.

REFERENCES

A1. P.W. Hannan, "The Element-Gain Paradox for a Phased Array Antenna," *IEEE Trans. Antennas Propagat.* **AP-12**, 423-433, July 1954.

A2. A. Hessel and J.C. Sureau, "On the Realized Gain of Arrays," *IEEE Trans. Antennas Propagat.* **AP-19**, 122-124, Jan. 1971.

A3. W.K. Kahn, "Efficiency of a Radiating Element in Circular Cylindrical Arrays," *IEEE Trans. Antennas Propagat.* **AP-19**, 115-117, Jan. 1971.

A4. W.K. Kahn, "Ideal Efficiency of a Radiating Element in an Infinite Array," *IEEE Trans. Antennas Propagat.* **AP-15**, 534-538, July 1967.